= Year 12 = Algebra of functions = Worksheet 1

1. For $f(x)=x+\frac{1}{x}$, show that (i) $f\left(\frac{1}{u}\right)=f(u)$,
(ii) $f(uv)+f\left(\frac{u}{v}\right)=f(u)f(v)$, for $u,v\in R\setminus\{0\}$.

2. For $f(x)=1-e^{x}$, show that (i) $f(u)f(-u)=f(u)+f(-u)$,
(ii) $f(u+v)=f(u)+f(v)-f(u)f(v)$, for $u,v\in R$.

3. For $f(x)=1-\sqrt{x}$, show that (i) $f\left(\frac{x}{y}\right)=\frac{f(x)-f(y)}{1-f(y)}$,
(ii) $f(xy)=f(x)+f(y)-f(x)f(y)$, for $x,y\in R^{+}$.

4. For $f(x)=1+\log_e x$, show that (i) $f(xy)=f(x)+f(y)-1$,
(ii) $f\left(\frac{x}{y}\right)=f(x)-f(y)+1$, for $x,y\in R^{+}$.

5. Refer to Q4. Show that (i) $f(xy)+f\left(\frac{x}{y}\right)=2f(x)$,
(ii) $f\left(\frac{x}{y}\right)-f\left(\frac{y}{x}\right)=2[f(x)-f(y)]$.

6. For $f(x)=1+x^{2}$, show that
(i) $f(x+y)+f(x-y)=2[f(x)+f(y)-1]$,
(ii) $f(x+y)-f(x-y)=4xy$,
for $x,y\in R$.

7. For $f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}$, show that for $x,y\in R^{+}$,
$f(x^{2})-f(y^{2})=[f(x)-f(y)].[f(x)+f(y)]$.

8. For $f(x)=e^{x}+e^{-x}$, show that
$f(2x)-f(2y)=[f(x)-f(y)].[f(x)+f(y)]$.

9. For $f(x)=e^{x}-e^{-x}$, show that (i) $[f(x)]^{3}=f(3x)-3f(x)$,
(ii) $[f(x)]^{5}=f(5x)-5f(3x)+10f(x)$.

10. For $f(x)=1-e^{-x}$, show that
$[f(x)]^{3}=3f(x)-3f(2x)+f(3x)$.